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Seiberg duality in three dimensions

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Abstract

We analyze three dimensional gauge theories with Sp gauge group. We find that in some regime the theory should be described in terms of a dual theory, very much in the spirit of Seiberg duality in four dimensions. This duality does not coincide with mirror symmetry.

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1. Introduction

The past few years have seen a tremendous advance in our understanding of supersymmetric gauge theories in various dimensions. Recently interest has focussed on $N = 2$ supersymmetric field theories in three dimensions [4,6,5]. Theories with double the amount of supersymmetry ($N = 4$) in three dimensions have allready been extensively studied and were found to posses a certain duality, called mirror symmetry [7,8]. It interchanges Coulomb and Higgs branch of the two theories which are mirror to each other.

Also many results have been obtained in theories with the same amount of supersymmetry, but with one more space dimension, that is $N = 1$ theories in four dimensions ([1,2]). These theories have a duality symmetry which is usually refered to as Seiberg duality. Two different gauge theories flow to the same interacting IR fixed point. It is an interesting question, which of those dualities survive the transition to $d = 3$, $N = 2$ and how they turn out to be related if they do. Some progress was achieved in [5,4,6] where it was shown that at least mirror symmetry has a counterpart in $d = 3$, $N = 2$ theories. Mirrors were explicitely constructed for abelian gauge groups. They were shown to have an interpretation as the theory of vortices in [6]. In [6,4] also the tools were developed for the study of the non-abelian theories. We will mostly stick to the notation of [6].

The question whether Seiberg duality survives dimensional reduction has not been answered so far. In this letter we will argue that there actually exists a corresponding duality in the three dimensional context. It basically leaves the Higgs/Coulomb structure untouched and hence does not coincide with mirror symmetry. We will explicitly construct this duality for $Sp(2N_c)$ [1] with $2N_f$ fundamental flavors, since this seems to be the easiest case to study. It can probably be generalized to other gauge groups as well. In section 2 we will describe the model, analyze its behaviour for small values of N_f following [6] and construct the dual. In section 3 we will describe some consistency checks: the dual model captures the structure of Coulomb and Higgs branch, reacts consistently under perturbations, reduces to the corresponding Seiberg duality constructed in [2] upon decompactification and satisfies the mod 1 parity anomaly matching conditions of [6]. In Section 4 we will describe the brane point of view and conclude.

2. The model

The model we study is very similar to $d = 3$, $N = 2$ SUSY QCD with $SU(n)$ gauge group which was extensively studied in [6]. We refer the reader to their paper for a discussion of $N = 2$ supersymmetry in 3 dimensions and the tools to analyze the corresponding quantum theories. We will be dealing with a $Sp(2N_c)$ gauge group instead. This has the advantage that the only gauge invariant operators we have to consider are the mesons. In Sp theories the baryons are just products of mesons. Let us first introduce the building blocks of our analysis: we have $2N_f$ quarks Q transforming as fundamentals of the gauge group[2]. The

[1] We will denote by $Sp(2N_c)$ the group whose fundamental representation has dimensoin $2N_c$

[2] Even though in three dimensons theories with an odd number of flavors would be anomaly free, too, with the addition of suitable Chern-Simons terms, we will only consider theories with an even number of flavors, since we do not want to consider nonvanishing Chern-Simons contributions.

only gauge invariant operator we can build from them are the flavor antisymmetric mesons:

$$M_{ij} = \omega_{ab} Q_i^a Q_j^b$$

where a, b are color indices, i, j are flavor indices and ω is the 2-index antisymmetric invariant tensor of Sp. M parametrizes the Higgs branch of our model.

In addition we will have to deal with a scalar ϕ transforming in the adjoint representation of the gauge group. It is part of the vector multiplet. If one thinks of $N = 2$, $d = 3$ as dimensional reduction of $N = 1$ in $d = 4$ ϕ is the component of the 4d vector in the reduced direction. If we are on the Coulomb branch, there is another scalar γ, which is the dual of the vector. They can be combined into a complex scalar $\Phi = \phi + i\gamma$. Let's write adjoint scalar in the form $\phi = diag(\phi_1, \dots, \phi_{N_c}, -\phi_1, \dots, -\phi_{N_c})$. By using the Weyl group we can take $\phi_1 \geq \phi_2 \geq \dots \geq \phi_{N_c} \geq 0$. Define semi-classically the fundamental instanton factors as

$$\begin{aligned} Y_j &\sim e^{(\Phi_j - \Phi_{j+1})}, \quad j = 1, \dots, N_c - 1 \\ Y_{N_c} &\sim e^{2(\Phi_{N_c})} \end{aligned}$$

Again we expect that for $N_f \neq 0$ (like in [6]) the Coulmob branch is lifted by instanton effects except for a one dimensional subspace. This is parametrized by an instanton factor $Y = \prod_{j=1}^{N_c} Y_j$ which is globally well defined. To see how Y transforms under the global symmetries one has to count fermionic zero modes. Using the formulas presented in [4] it is easy to verify that Y picks up a N_c factors of r-charge -2 from the gaugino zero modes in the N_c fundamental instantons. In addition Y_{N_c} and hence also Y picks up one fermionic zero mode from every massless quark present. To summarize this, the following table shows these building blocks and how they transform under the global symmetries. Note that there are no anomalies for the global symmetries in 3 dimensions, so we have the full classical symmetry.

	$U(1)_R$	$U(1)_A$	$SU(2N_f)$
Q	0	1	$\mathbf{2N_f}$
M	0	2	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$
Y	$2(N_f - N_c)$	$-2N_f$	$\mathbf{1}$.

The behaviour of the quantum symmetry depends crucially on the number of flavors:

2.1. $N_f = 0$:

With no massless quarks present, all the fundamental instantons have just r-charge -2. A superpotential

$$W = \sum_{j=1}^{N_c} \frac{1}{Y_j}$$

is created, where Y_j denotes the jth fundamental instanton. The full Coulomb branch is lifted and there is no stable supersymmetric groundstate.

2.2. $0 < N_f < N_c$:

Since now one fundamental instanton picks up a quark zero mode contribution and hence can not appear in the superpotential, we expect a one dimensional sublocus of the Coulomb branch (parametrized by Y) to remain unlifted. In terms of Y and M a superpotential is created, which is completely characterized by the charge assignments (up to a normalization):

$$W = (N_c - N_f)(Y\mathrm{Pf}\ M)^{1/(N_f - N_c)}.$$

Again we are left without a stable vacuum.

2.3. $N_f = N_c, N_c + 1$:

For $N_f = N_c$ a quantum constraint on the moduli space is consistent with the global symmetries:

$$Y\mathrm{Pf}\ M = 1.$$

The moduli space is a merged Higgs and Coulomb branch.

For $N_f = N_c + 1$ the theory confines and the relevant degrees of freedom are Y and M interacting via a superpotential:

$$W = -Y\mathrm{Pf}\ M.$$

In the 4d context this kind of behaviour was called s-confining ([3]). All this is very similar to the phenomena observed for $SU(n)$ gauge group in [6]. In their case the s-confining superpotential contained a cubic term involving the baryons and hence led to a non-trivial fixed point. The absence of the baryons makes the physics of the Sp groups somewhat easier to study.

Upon decompactification to 4 dimensions (which corresponds to adding a term ηY to the superpotential, with $\eta \sim e^{-1/Rg_3^2}$ ([6])) these results nicely connect to the corresponding theories studied in [2] (in the 4d limit $\eta \sim e^{-1/g_4^2} \sim \Lambda^{\beta_0}$, $\beta_0 = 3\mu_{gauge} - \mu_{matter}$ is the one loop beta function). Note however that the sequence of the 3d theories is shifted by one from the corresponding 4d theories (there one has: no ground state for $N_f \leq N_c$, quantum constraint for $N_f = N_c + 1$ and s-confining for $N_f = N_c + 2$). For $N_c = 1$ the picture described so far coincides with the $SU(2)$ analysis of [6].

2.4. $N_f \geq N_c + 2$

In the spirit of [6] we assume the following structure emerging as we drive the theory towards higher values of N_f. There will be a 1-dimensional Coulomb branch parametrized by Y intersecting the Higgs branch at $Y = 0$. At the intersection we get an interacting SCFT, best described by some new degrees of freedom. We claim that in fact this setup has a dual description, very much like the ones found in the 4 dimensional context. The dual theory has gauge group $Sp(2(N_f - N_c - 1))$ and the following content:

	$Sp(2(N_f-N_c-1))$	$U(1)_R$	$U(1)_A$	$SU(2N_f)$	$SU(2)$
q	$\square$	1	-1	$\mathbf{2N_f}$	
t	$\square$	N_c-N_f+1	N_f	$\mathbf{1}$	2
M	1	0	2	$\begin{smallmatrix}\square\\\square\end{smallmatrix}$	
Y	1	$2(N_f-N_c)$	$-2N_f$	$\mathbf{1}$.	

and an aditional superpotential

$$W = Mqq + Ytt + \tilde{Y}.$$

$\tilde{Y}$ is the corresponding object to Y of the magnetic theory. It picks up global charges from gaugino zero modes of the dual group and quark zero modes of q and t. The superpotential removes the Coulomb branch associated with the magnetic gauge group. Note that the appearence of $\tilde{Y}$ in the superpotential reduces the number of global $U(1)$s by one (as the other two terms naturally do, too). This leaves us indeed with the same global symmetries as we had in the original model. The fields M and Y appear as fundamentals in the dual theory, the magnetic invariants qq and tt get removed via the superpotential. Probably we should think of the internal $SU(2)$ as being gauged, too, so that there are no invariants qt. In the following we will present some evidence that this dual picture indeed captures the physics of our original problem.

3. Consistency Checks

3.1. The parity anomaly matchings

As pointed out in [6] there is something similar to the 't Hooft anomaly matching conditions in three dimensions: whether or not a weakly gauged global symmetry would require a Chern-Simons term and hence break parity should match in both theories. Since this is only a yes or no question (which amounts on checking if the one-loop contribution is integer or half-integer), this check is much weaker than the 4d equivalent, but nevertheless provides us with some method to check our claims. In both theories we find that the corresponding Chern-Simons terms have coefficients (mod 1): $k_{RR} = \frac{N_c}{2}$, $k_{AA} = 0$, $k_{SU} = 0$

3.2. Classical Constraints on Higgs and Mixed branch

Classically M is constrained to have rank $\leq 2N_c$. On the magnetic side M appears as a singlet field and is classically unconstrained. For nonzero Y in general we can still have nonvanishing M giving rise to mixed branches. In this case $rank(M) \leq 2(N_c-1)$, which amounts to the classical condition of having at least one unbroken $U(1)$. It is very convincing to see how these constraints arise through quantum effects in the dual description.

First consider the case $Y \neq 0$. The superpotential gives mass to t and we are left with an $Sp(2(N_f-N_c-1))$ gauge theory with the $2N_f$ fundamentals q and the superpotential $W = Mqq + \tilde{Y}$. Giving a vev to M gives a mass to $\frac{rank(M)}{2}$ of the q. If $rank(M)$ is to high the number of massless flavors drops into the regime where we generate a superpotential and

remove all supersymmetric ground states. This puts a quantum constraint on the rank of M. So far this discussion is identical to the $d=4$ case. In what follows we will see that the $\tilde{Y}$ term plays a crucial role and the consistency of the construction strongly supports the $d=3$ picture. The Mqq term in the superpotential tells us that the only possible ground state is at the origin, $q=0$. In our previous analysis we found that if N_f^L (number of massless quarks, $N_f - \frac{rank(M)}{2}$) is less or equal than $\tilde{N}_c (= N_f - N_c - 1)$, the origin is removed from the quantum moduli space. For $N_f^L = \tilde{N}_c + 1$ we found that the quantum moduli space is the same as the classical moduli space described in terms of $\tilde{Y}$ and Pf q^2 interacting via a superpotential:

$$W = \tilde{Y} \mathrm{Pf}\ q^2.$$

and hence includes the origin. With the additional superpotential term $\tilde{Y}$ this becomes

$$W = \tilde{Y}(\mathrm{Pf}\ q^2 + 1)$$

which again gives a smoothed out moduli space and removes the origin. Hence we arrive at the following quantum constraint:

$$\begin{aligned} N_f^L &\geq \tilde{N}_c + 2 \\ rank(M) &\leq 2(N_c - 1) \end{aligned}$$

exactly reproducing the classical constraint of the original theory.

For $Y = 0$ we have an additional massless flavor, t, and hence the above analysis gets modified to yield $rank(M) \leq 2N_c$, which again agrees with the classical constraint obtained before.

3.3. Decompactification to 4 dimensions

In [6,13] it was argued that decompatification to 4 dimensions can be achieved by taking into acount $d=3$ instantons which are related by a large gauge transformation around the compact circle to the usual instantons. Their effect is included by adding to the superpotential a term ηY where $\eta \sim e^{-1/Rg_3^2}$ which becomes 0 in the 3d limit and Λ^{β_0} in the 4d limit. Adding this terms to the dual theory the equations of motion for Y tell us that:

$$< tt > = -\eta.$$

This breaks the internal $SU(2)$ and higgses the dual gauge group to $Sp(2(N_f - N_c - 2))$, which is precisely the expected magnetic gauge group found in 4 dimensions ([2]). The t fields get eaten by the Higgs mechanism and the $\tilde{Y}$ term in the superpotential gets removed by considering the effects of decompactifying the magnetic gauge dynamics. Hence we are only left with an 4 dimensional $Sp(2(N_f - N_c - 2))$ gauge theory with $2N_f$ quarks q and gauge singlets M interacting via a superpotential.

$$W = Mqq.$$

The scale of the magnetic theory after the higgsing is $\tilde{\Lambda} \sim \frac{1}{<tt>} \sim \frac{1}{\eta} \sim \frac{1}{\Lambda^{\beta_0}}$. Our model reduces exactly to the well known Seiberg duality in 4 dimensions.

3.4. Reduction to known results

The model should be consistent under perturbations. While adding real quark masses yields a rather complicated structure, the effect of complex masses is easily studied. In fact it is exactly the same as in 4 dimensions, since M only interacts via the Mqq term in the superpotential. Similarly perturbations along the M flat directions can be analyzed. Note that other than in 4 dimensions the cubic superpotential alone guarantees that we get a non-trivial fixed point. We don't get a mapping to free magnetic theories.

To get a new consistency check it is instructive to study the breaking of $N_f = N_c + 2$ to $N_f = N_c + 1$. The dual group is completely broken. Instanton contributions should reproduce the result presented in section 2. Consider adding to the $N_f = N_c + 2$ theory a superpotential $W_{tree} = mM_{N_f+2,N_f+2}$. Assume that the light mesons $\hat{M}$ are all nonzero. This leaves the dual $Sp(2)$ theory with two quarks q and t, where $Sp(2)$ is then broken by the non-zero $< qq >= -m$. At this stage the superpotential reads

$$W = \tilde{Y} tt < qq > + Y_L tt + \tilde{Y}$$

yielding a vev of $< tt >= -\frac{1}{<qq>} = \frac{1}{2m}$ (breaking the internal $SU(2)$). Since Y_L can be associated with the scale of the low energy theory (after integrating out the quarks that get masses $< \hat{M} >$) we expect $Y_L \sim Y \mathrm{Pf} \ < M >$. Finally we reproduce as expected the $N_f = N_c + 1$ superpotential

$$W = Y \mathrm{Pf} \ \hat{M}$$

4. Brane Interpretation and Conclusions

In the brane setup this theory can be obtained by rotating a NS5 brane in the Hanany-Witten setup [8] or by T-dualizing the EGK [9] setup. The first approach corresponds to breaking $N = 4$ to $N = 2$ and the second one to reducing $N = 1$, $d = 4$ down to three dimensions. Viewing things this way makes mirror symmetry a natural consequence of the S-duality of Type IIB. On the other hand Seiberg duality was obtained in a rather complicated fashion by pushing around the branes of EGK. As discussed in [10] it is not even clear if this really derives Seiberg duality from brane dynamics. It is not possible to decide from this description, whether Seiberg survives reduction to 3d or not. But at least we would not expect that Seiberg duality should reduce to mirror symmetry, since latter is allready taken care of by S-duality. This is consistent with what we presented in this paper.

The brane picture may also explain, where the shift in the dual gauge group comes from (note that instead of $Sp(2(N_f - N_c - 2))$ in 4d we have $Sp(2(N_f - N_c - 1))$ in 3d). In 4d the -2 was obtained in the various approaches [11,10] by pushing branes through orientifold planes. Charge conservation required the destruction of two branes in certain transitions to cancel the charge of the orientifold, giving rise to the above -2 factor. Going to 3 dimensions via T-duality reduces the orientifold by one dimension and hence halves it charge ([12]), substituting -2 with -1.

We expect similar dualities to be valid for other gauge groups, too. Their structure is certainly more complicated due to the presence of baryons. We hope to be able to report on progress in those direction soon.

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